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Anomalous Hall effect in a wide parabolic well

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We report observation of the anomalous Hall effect in a wide $Ga_{1-x}Al_xAs$ parabolic wells in quasi-parallel magnetic field. We found that the slope of the Hall resistance at high magnetic field B is two time larger than the slope at low B and depends on the temperature. The anomalous Hall effect can be attributed to the unusual spin properties in parabolic wells, for example, the effective g-factor in these structures changes the sign along well width. It may suppress the motion of electrons in the crossed electric and magnetic fields.

1 Introduction

Anomalous Hall effect has been discovered almost 50 years ago. It has been intensively studied in ferromagnetic systems and it was proposed that it originates from spin–orbit and exchange interactions [1]. It was found that the Hall resistivity is larger than in nonmagnetic metals and can be fitted empirically by the formula

$$R_{xy}= R_0B + R_SM \tag{1}$$

where B is applied magnetic field, R_o is the ordinary Hall coefficient , and R_S is the anomalous Hall coefficient. In the free electron theory of the conductivity the normal Hall coefficient is given by simple equation:

$$R_0= -1/en \tag{2}$$

where n is the density of electrons and e is electronic charge, and as we can see it depends only on the electron concentration. The magnitude of the anomalous Hall coefficient depends on the various parameters of the material and its structure. For example, the magnitude of R_S is extremely large in amorphous ferromagnetics, it can be larger than R_0 by a factor of a hundred to a thousand, and may have the opposite sign. The characteristic curve for the Hall effect in typical ferromagnet films has the following behaviour. For low magnetic field the perpendicular component of the magnetization equals to the applied magnetic field, therefore the Hall resistivity increases linearly with applied field with slope, which is much larger, than expected from equation (2). When the magnetic field exceeds the saturation magnetization, the perpendicular component of the magnetization is constant, which leads to the constant anomalous Hall resistivity. However the total Hall resistivity is not saturated at this field, it demonstrates further increasing with magnetic field due to the ordinary Hall effect. Since the slope of the normal Hall effect is usually smaller than the magnitude of R_S , the curve shows the kink in magnetic field, when B

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reaches the value of the saturation magnetization. If R_0 has the opposite sign to R_S, the Hall resistance decreases after the kink.

The asymmetric scattering of electrons by magnetic atoms may lead to the anomalous Hall voltage in ferromagnetic samples. It is now accepted that two main mechanisms are responsible for such effect: the skew-scattering proposed in work [2], and the side-jump proposed in [3]. It is worth noting that in these models the carriers are assumed to be magnetic and the scattering centers non-magnetic. Indeed the result should be the same, when the situation is reversed. For skew-scattering model the plane wave is scattered by impurity in the presence of the spin–orbit coupling. In such situation the amplitude of the wave packet becomes anisotropic and depends on the spin. The average trajectory of electron is deflected by a spin dependent angle, which is typically of order 10^{-2} rad. For the side jump mechanism the center of the wave packet is shifted during the scattering, and this shift is also spin dependent. The typical lateral displacement during this process is 10^{-13} cm. It is worth noting that the deflection of the electron trajectory is very small, however we can explain the large magnitude of R_S in amorphous ferromagnet by large number of scatterers, since practically every atom scatters the electron.

Until recently anomalous Hall effect has been intensively studied only in magnetic structures. In non-magnetic semiconductors the anomalous Hall effect is very small, and can be separated from ordinary Hall effect by magnetic resonance of the conduction electrons [4]. In InSb the Hall angle was found $3x10^{-4}$ rad, which is 4 order of magnitude smaller, than angle for normal Hall effect. In present work we report observation of the anomalous Hall effect with the same magnitude as an ordinary Hall effect in a nonmagnetic wide $Ga_{1-c}Al_cAs$ parabolic wells in quasi-parallel magnetic field.

2 Electronic states and spin polarization of a wide parabolic well in magnetic field

The first wide parabolic wells were grown by Sundaram [5] and by Shayegan [6]. The parabolic variation of the well potential was introduced to avoid soft barrier in the center originating from Coulomb interaction among electrons in the wide quantum well. It allows to create almost square wide well, when this well is filled by elelctrons. Figure 1 shows the variation of Al composition along z-axis. Finally the two-dimensional electron system with several subbands occupied has been obtained.

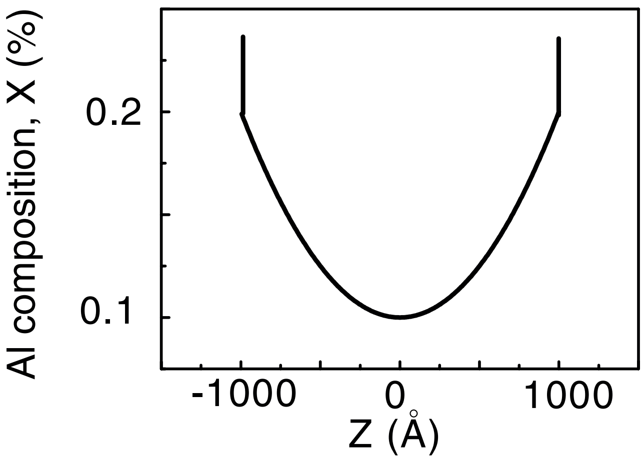

Fig. 1 The variation of the Al composition in 2000 Å parabolic well along Z axis.

The problem of quasi-two-dimensional electrons in tilted magnetic field gas been solved analytically for parabolic well in [7]. The energy of the electrons in a parabolic quantum well with the potential $V = (az)^2$ in tilted magnetic field is given by:

$$E = E_\alpha(n_\alpha + 1/2) + E_\beta(n_\beta + 1/2), \tag{3}$$

$$E_\alpha = [(\hbar\omega_c\cos\alpha)^2 + (\hbar\Omega\sin\alpha)^2 - (\hbar\omega_c\hbar\Omega)\sin2\alpha\sin\theta]^2 , \quad (4)$$

$$E_\beta = [(\hbar\omega_c\cos\alpha)^2 + (\hbar\Omega\sin\alpha)^2 + (\hbar\omega_c\hbar\Omega)\sin2\alpha\sin\theta]^{2,} \quad (5)$$

where $\Omega = a(2/m)^{1/2}$, $\omega_c = eB/mc$, m is the effective mass, n_α and n_β are integers and θ is the tilt angle between the magnetic field and the normal to the parabolic well plane z. The angle rotation α can be obtained from another equation:

$$\tan2\alpha = (\hbar\Omega)(\hbar\omega_c)\sin\theta /[(\hbar\Omega)^2 - (\hbar\omega_c)^2] \quad (6)$$

Figure 2 demonstrates evolution of the Landau levels (LL) belonging to the different subbands for W_e = 800 Å parabolic well in perpendicular magnetic field to the bulk Landau levels in parallel magnetic field, when B is tilted away from z (neglecting spin splitting). We should emphasize here that the levels in the strong magnetic field consists of the electronic subbands belonging to n = 0 Landau level. The energy spacing is described by the formula:

$$\Delta E = \hbar\Omega\cos\theta \quad (7)$$

and therefore diminishes with tilt angle. However, full PQW resembles rather square quantum well than harmonic potential. In this case the energy spectrum in zero magnetic field E_i of a PQW can be roughly approximated by spectrum of square well $E_i = i^2(h/W)^2/8m$, of width equal to that of the electron layer. In this case the energy level structure in the tilted magnetic field can not be obtained analytically. However in the limit of the strong magnetic field, where the magnetic length $l_B = (\hbar\ c/eB)^{1/2} << W_e$, the energy spacing proportional to $(\cos\theta)^2$ [8].

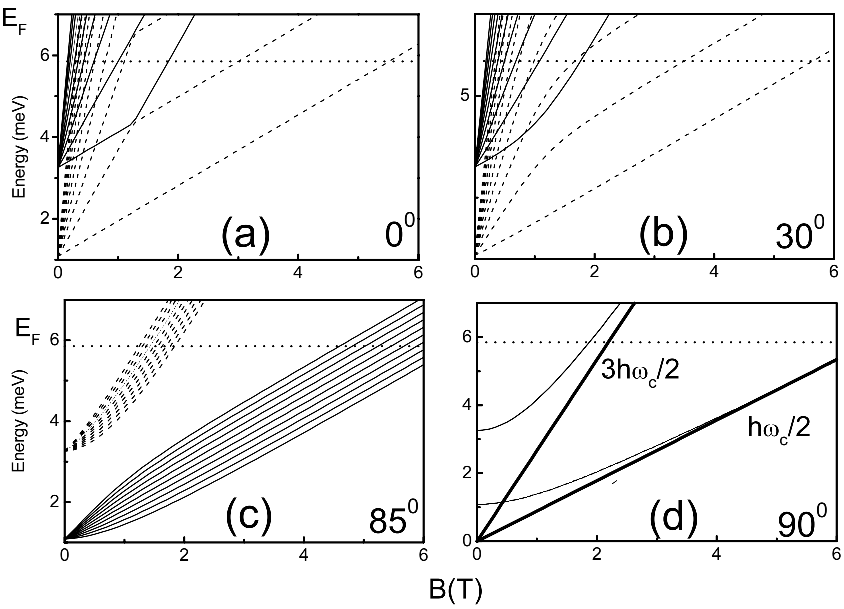

Fig. 2 Energy of a wide 2000 Å parabolic well as a function of the magnetic field for different tilt angles Θ. Ten Landau levels for each subband are shown. Spin splitting is neglected.

Figure 2 shows that in quasi-parallel magnetic field two-dimensional Landau states are collapsed into 2 three dimensional Landau state. However, the degeneracy of the oblique states depends only on the magnetic field component perpendicular to the quantum well plane and equal to $eB_\perp/hc$. Therefore it is expected that at the large tilt angles in the strong magnetic field corresponding to the Landau filling factor ν = 1,2 quantum Hall effect behaviour resembles to the behaviour in perpendicular magnetic field, if the only normal component of the field is taken into account. Indeed the three-dimensional limit may be reached when $\Theta \to 90^0$ and the distance between two-dimensional Landau subbands $\Delta E \to 0$, as can be seen in Fig. 2d. In a real system the energy levels have a finite width due to the disorder γ, therefore the electron system has a 3D energy spectrum when the electron subbands overlap, $\gamma \sim \Delta E$, which probably occurs in the interval $\Theta \sim 85^0$–90^0 .

Let us focus on the spin properties in the wide parabolic wells. The effective Landé g-factor in $Ga_{1-x}Al_xAs$ materials depend strongly on the Al composition x. As we can see in Fig. 1, if z = 0 is the position of the pure GaAs material, and effective harmonic potential is given by $V = m\Omega^2 z^2/2$, then a composition profile $c(z) = bz^2$ is achieved. The effective g-factor changes with composition: $g(x) = -g_0 + g_1 x$ [9], where g_0=0.44, and g_1 = 2.7. Therefore, g-factor increases monotonically from g = –0.44 (middle of the well) to g = +0.4 at the edge of the well (x = 0.3), and changes the sign at x = 0.13. As a function of the position z in the well g-factor is given by: $g(z) = -g_0 + ag_1 z^2$. Figure 3 shows the variation of the g-factor in parabolic well along z axis.

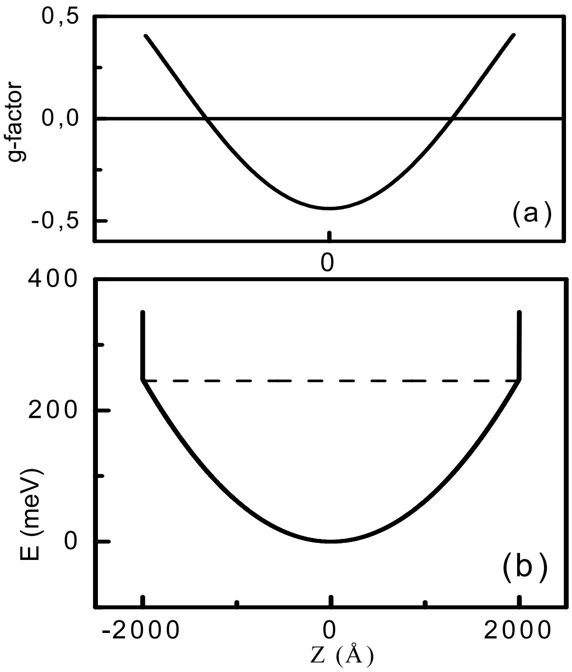

Fig. 3 The effective g-factor (a) and variation of the conductivity band (b) along z direction for 4000 Å wide parabolic well .

In magnetic field for harmonic potential the energy spectrum can be calculated exactly, and due to the quadratic dependence of the g-factor on z, one of the natural frequency is normalized by the spin factor [10]. In strong parallel magnetic field, when the cyclotron diameter becomes smaller than the well width, the spectrum turns on the spectrum of the three-dimensional gas (Fig. 2d) in the quantum limit with g-factor variable along z.

3 Experimental results and discussion

The samples were made from $Al_xGa_{1-x}As$ parabolic quantum well grown by molecular-beam epitaxy. It included a 1500–4000 Å-wide parabolic $Al_xGa_{1-x}As$ well with x varying between 0 and 0.29, bounded by undoped Al_yGa_yAs spacer layers with δ-Si doping on two sides [11]. The mobility of the electron gas in our samples was ~200–300 x10^3 cm^2/Vs and density – (3–4)x10^{11} cm^2, therefore our quantum wells were partially full with 3–5 subbands occupied. The test samples were Hall bars with the distance between the voltage probes L = 200 µm and the width of the bar d = 100 µm. Four–terminal resistance R_{xx} and Hall R_{xy} measurements were made down to 1.5 K in a magnetic field up to 12 T.

Figure 4 shows longitudinal R_{xx} and Hall R_{xy} resistances of 1500 Å-wide parabolic $Al_xGa_{1-x}As$ well at (=89.60 as a function of applied magnetic field for different temperatures. We may see that the magnetoresistance reveals oscillations, sometimes called diamagnetic Shubnikov de-Haas (SdH) oscillations, which results from the combined effect of the electric and magnetic fields. In quantum well with several subbands occupied such oscillations are interpreted as magnetic depopulation of the two-dimensional levels. Figure 4 shows 4 oscillations, therefore 4 subbands are depopulated in this 1500 Å -wide parabolic Al_xGa_{1-x} As well. The curve 1 is measured for the experimental geometry, when magnetic field is directed perpendicular to the current, and curve 2 is measured, when magntic field was oriented parallel to the current flow. We may see also that the amplitude of the magnetooscillations for curve 1 is larger than for curve 2.

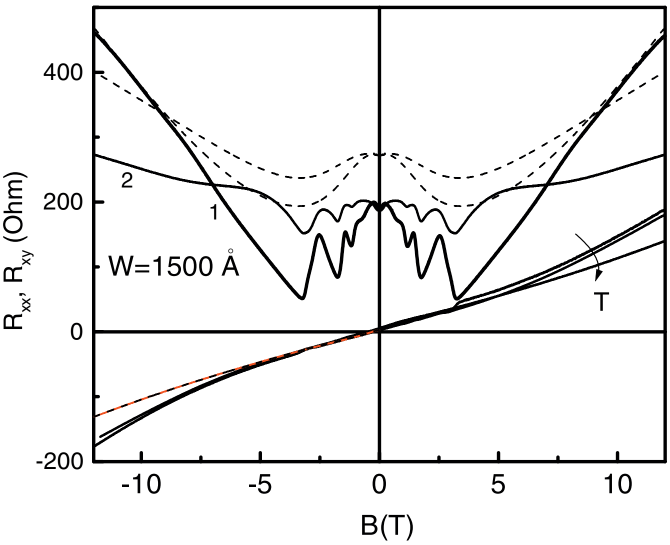

Fig. 4 The longitudinal and Hall resistance as a function of magnetic field at $\Theta = 89.6^0$ for different temperatures (T: 1.5 K, 4.2 K, 20 K, 30 K). Curve 1 shows magnetoresistance , when magnetic field is directed perpendicular to the current flow. Curve 2 shows magnetoresistance , when magnetic field is directed parallel to the current flow. Dashes –magnetoresistance at T = 20 K and 30 K.

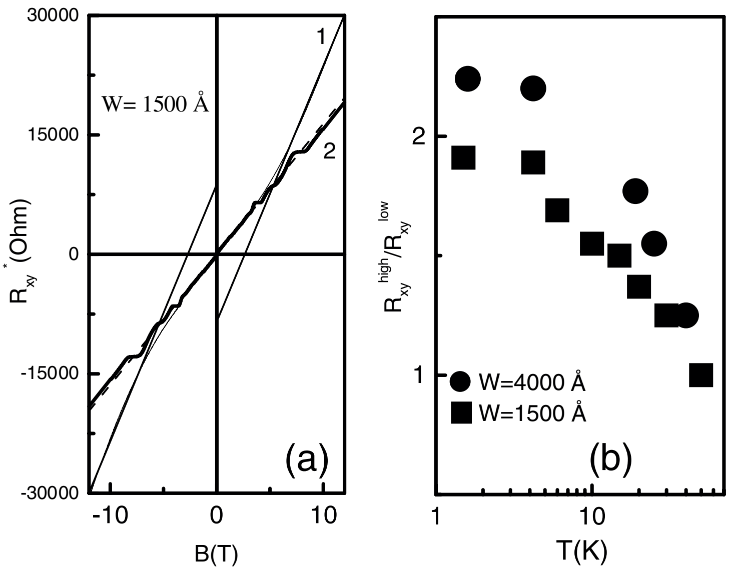

Fig. 5 (a) Normalized Hall resistance at $\Theta = 89.6°$ (1) and at $\Theta = 0°$ (2) as a function of magnetic field, T = 1.5 K. Dashes are the normalized Hall resistance at $\Theta = 89.6°$ as a function of magnetic field at T = 50 K. The high field part of the curve 1 is prolonged until it cross the y axis. (b) The ratio between the slope of the Hall resistance at low and high magnetic field as a function of temperature for two samples.

When the temperature increases, the oscillations are smeared out, and only minimum corresponding to the depopulation of the last Landau level is seen. The Hall resistance demonstrates linear dependence on the magnetic field at low B, and at higher field , R_{xy} deviates from the former linear dependence, and its slope increases. Such anomalous behaviour is observed only at low temperature, at T = 30–50 K, R_{xy} is recovered and demonstrates normal behaviour.

Figure 5a shows the Hall resistance in details. At relatively high (T = 20–30 K) temperatures in quasi-parallel magnetic field we observed linear Hall effect (dashes in Fig. 6a), described by conventional equation

$$R_{xy}(\Omega) = B_{\perp}(T)x10^{-4} /en_s(10^{11}cm^{-2}),$$

where $e = 1.6x10^{-19}$ C. At low (T = 1.5 K) temperature we found, that for magnetic field B below 5.2 T Hall resistance is not changed, however for B > 5.2 T it demonstrates unusual behaviour (curve 1 in Fig. 6a), which may be described by equation

$$R_{xy}(\Omega)/\sin\Theta = Ax10^{-4}(B_{\perp}(T)-5.2)/en_s +0.32h/e^2 .$$

The coefficient A gradually increases with temperature decrease and becomes 2 times larger at T= 1.5 K than at low field and high temperatures (Fig. 5b). In perpendicular magnetic field we also observed ordinary Hall effect (the curve 2 in Fig. 5a), however the energy spectrum transforms to Landau levels, and we may see the quantum Hall effect (plateaux in curve 2). It is worth noting that the curves in Fig. 6a at tilt angle $\Theta = 89.6^0$ are divided by factor sinΘ, since the Hall resistance is sensitive only to the normal component of the magnetic field. Therefore we may conclude that the anomalous Hall effect is observed only in quasi-parallel magnetic field and low temperatures.

We should mention that in our case the behaviour of the Hall resistance is different from the Hall effect in ferromagnets: the slope of the Hall resistance at higher field is larger than the slope at low B and depends on the temperature. In ferromagnets the slope at higher field is determined by the ordinary Hall coefficient and usually it is smaller than anomalous slope at low B.

We attribute the anomalous Hall effect in our parabolic well to the unusual spin properties. The effective g-factor in such structure changes the sign along well width, as we demonstrates above (see Fig. 3a). In strong quasi-parallel magnetic field magnetic length becomes smaller than the sample width. In this case the states in the different parts of the well along z axis has the different spin polarization: the center of the well is almost antialigned spin and the edge of the well is almost all aligned spins. It is worth noting that in nonmagnetic material there is no spontaneous magnetization, and electrons can be polarized by application of the external magnetic field. Since spin-dependent Hall effect is very small, it is difficult experimentally to separate anomalous and ordinary Hall effect in semiconductors. In paper [4] such separation has been done by application of the microwave irradiation keeping the applied magnetic field constant. In this case the ordinary Hall effect remains constant, spin polarization decreases, which leads to the variation of the spin-dependent Hall effect. However, these variations were 4 order of magnitude smaller, than the ordinary Hall effect. In our parabolic well the anomalous Hall effect has the same magnitude, as a normal Hall effect. We attribute the large Hall coefficient in our system to the suppression of the electron motion in crossed electric and magnetic fields. The ordinary Hall effect arises from the Lorentz force that acts on a moving charge. In the tilted magnetic field electron in parabolic well moves in y and z direction (when field is tilted in y direction, and current flows along x axis). However, the motion in z direction requires spin flip process, which are suppressed in the case of the small spin-orbit interaction. Effectively the number of electrons, which moves in the direction perpendicular to the current and magnetic field decreases, and , as we can see for equation (2), Hall coefficient increases.

In conclusion we demonstrate that remotely doped $Ga_{1-x}Al_x$ As parabolic quantum well is promising system providing effective control and manipulation with the electron spin, because the spin properties of such materials depend strongly on the Al composition x. The effective g-factor in such structure changes the sign along well width, which may lead to the suppression of the classical motion in crossed electric and magnetic field. We found that the variation of the g –factor affects the electron dynamics and the Hall resistance and leads to the anomalous Hall effect. We measured this effect in different parabolic wells as a function of the temperature and the tilt angle.

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